Effective Field Equations of Brane-Induced Electromagnetism

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Abstract

Using a covariant embedding formalism, we find the effective field equations for the Electromagnetism that emerge on branes in the context of Dvali-Gabadadze-Porrati (DGP) braneworld scenario. Our treatment is essentially geometrical. We start with Maxwell equations in five-dimensions and project them into an arbitrary brane. The formalism is quite general and allows us to consider curved bulk spaces and curved branes whose tension is not necessarily null. The kinetic electromagnetic term induced on the world volume of the brane, proper of DGP models, is incorporated in this formulation by means of an appropriate match condition. We also give an estimate of each term of the effective field equations and determine the domain in which the four-dimensional Maxwell equations can be recovered in the brane.

1 Introduction

According to recent braneworld scenarios all fields of the Standard Model are trapped in a four-dimensional (4D) spacetime, which is seen as a manifold embedded in a higher-dimensional space [1, 2, 3, 4]. In this picture the ambient space, or *bulk*, appears to be more fundamental, in a sense,

than the ordinary four-dimensional spacetime. Therefore it seems plausible that the basic laws of physics should primarily be formulated in this higher-dimensional setting. According to this view, the apparent validity of our familiar four-dimensional laws would merely appear as a consequence of more fundamental higher-dimensional laws [5, 6, 7, 8]. This is very much in the spirit of the original Kaluza-Klein theory and its modern versions where the fundamental field equations, which unify gravity and electromagnetism, are formulated in a higher-dimensional manifold [9]. Another proposal that essentially follows the same principle is known as the induced-matter approach [10, 11, 12]. Here the fundamental field equations are again five-dimensional (5D) and four-dimensional macroscopic matter configurations are regarded as a purely geometrical manifestation of the extra dimension. Of course the "embedding" of a four-dimensional theory into five or higher dimensions is naturally subjected to the so-called embedding theorems [13]. In the case of the induced-matter approach the embedding is guaranteed by the well-known Campbell-Magaard theorem [14, 15, 16, 17, 18, 18, 20], whereas in the case of branes an extended version of the latter is the relevant mathematical result to be used [21, 22].

A fundamental ingredient in the brane world scenarios is the mechanism of field localization. In the context of string theories these mechanisms arises very naturally [23]. But, there also exist methods of localization based on field-theoretic framework such as the capture of fermions by topological defects [24, 25]. In this paper we are going to consider localization mechanisms of vector fields provided by field-theoretic means. However, in a sense, the confinement of massless gauge fields requires a more intricate mechanism than those methods applicable to matter fields. The reason is that gauge fields trapped by topological defects will not be massless unless new elements be added to the theory [26]. Furthermore, as it was pointed out in Ref. [27], the confinement of gauge fields should preserve the universality of charge in the brane, but a hypothetical mechanism that was based on a direct localization of the zero mode in topological defects would have little chance to fulfill this principle.

There are alternative approaches which avoid these problems satisfactorily. Among them, we can mention models in which the confinement is performed by gravity [28, 29]. As a matter of fact, if the bulk is a five-dimensional warped space, it is known that gravity is not strong enough to keep the vector field localized in the brane [30, 31]. However, the confinement can work appropriately by adding new warped extra dimensions which

are also compact. Other models follow a complete different approach. They propose a modification of the gauge field equations in the bulk in order to achieve the localization in the brane [26, 32, 33, 34, 35].

In this paper we are going to focus our discussion on the well-known DGP model [36], because of its generality (it can be employed in any theories with extra-dimensions) and interesting physical implications, specially, for cosmology [37] (such as the possibility of explaining the late accelerated phase of expansion of the Universe). The appearance of DGP model might be seen as a further step in the road of extra dimension theories. Indeed, roughly speaking, we might say that the history of these theories begins with Kaluza-Klein's idea of tiny compact dimensions and develops towards the RSII model of infinite large dimension [4]. It turns out that within the RSII model the proper world volume integrated along the extra dimension is finite because of the warping factor. With DGP model this last constraint is abandoned and we are led to a consistent large extra dimension model which is also world volume infinite [36].

The basic assumption of DGP model is that radiative quantum corrections associated to the interactions between the bulk gauge fields and localized matter in the brane might be effectively incorporated in the theory by adding a new term in the Lagrangean, which corresponds to a kinetic term for the bulk fields, restricted to the world volume of the brane.

Our main goal in this paper is to obtain a covariant formulation of the effective equation for the electromagnetism in DGP scenario. In other words, we are interested in finding out how 4D-observers describe the behavior of the electromagnetic field, considering that they live confined in a brane (with a not necessarily null tension) in the context of DGP scenario. We will start with a five-dimensional version of Maxwell equations, which we will assume to hold in the bulk, and by using the formalism of embedding theory we will determine the induced equation in the brane. The covariant formalism used here is very general and embodies a great variety of geometric configurations. Indeed, except for the reflection symmetry, there is no restriction imposed on the geometry of the bulk and brane's manifolds.

As we shall see, the effective field equations contains additional terms as compared to the usual Maxwell equations and possess two important parameters: the brane tension (λ) and the five-dimensional electromagnetic coupling constant. We are going to suggest a possible physical interpretation for the new terms from the point of view of the 4D-observers and give an estimate for them relatively to the parameters.

By considering the electromagnetic coupling constant, the ratio between the four-dimensional and the five-dimensional values establishes a length scale in the DGP model (r_c) [38]. A very peculiar feature of DGP model is the prediction that deviations from the four-dimensional behavior occurs at ultra-large distances as compared to r_c, unlike other extra dimension theories.

In a brane with a non-zero tension, there will be two length scales regulating the effective equations: $(G_5\lambda)^{-1}$ and r_c, where G_5 is the five-dimensional gravitational constant. Of course, the induced equations on the brane should be consistent with all experimental data related to electromagnetic phenomena. Based on our estimates we are going to study the dependence of the effective field equations on these parameters and check under what condition Maxwell equations could be recovered.

The paper is organized as follows. In Section 2 we present the basics of the embedding formalism and the techniques of projecting general tensors on hypersurfaces. In Section 3, we establish a relation between the intrinsic and the projected electromagnetic field in a hypersurface, by means of which we find the effective equations in the hypersurface. We proceed with Section 4, where we discuss the question whether any solution of the 4D Maxwell equations might be reproduced from five dimensions. In section 5, assuming the reflection symmetry and using the appropriate match condition we obtain the effective field equations for the electromagnetic field in DGP scenario. Here we also discuss the low-energy limit of the effective equations. Finally, Section 5 contains our final remarks.

2 The embedding formalism

In this section we shall employ some techniques from differential geometry in order to set the basic equations that regulates the embedding of the four-dimensional electromagnetic equations in higher dimensions. We start by recalling some definitions.

Let us assume that our n-dimensional spacetime Σ is embedded into a space $\hat{M}$ with $(n+1)$ dimensions and let $\phi : \Sigma \rightarrow \hat{M}$ be the embedding map. Then, by definition of embedding, the image $\phi(\Sigma)$ is a hypersurface of $\hat{M}$ [39]. Now, if Σ and $\hat{M}$ are equipped with the metrics g and $\tilde{g}$, respectively, then the embedding is said to be isometric if the intrinsic metric of Σ corresponds

to the induced metric in the hypersurface, i.e., if the condition

$$g(\mathbf{v},\mathbf{w}) = \hat{g}\left(d\phi(\mathbf{v}), d\phi(\mathbf{w})\right) \tag{1}$$

holds for any vectors $\mathbf{v}$ and $\mathbf{w}$ of the tangent space $T_p\Sigma$ for every $p \in \Sigma$ [1]. Since ϕ is a homeomorphism let us, henceforth, identify Σ with $\phi(\Sigma)$ and as the differential map $d\phi$ is injective there is no confusion if we write simply $\mathbf{v}$ in place of $d\phi(\mathbf{v})$. Now let $\left\{y^A\right\}$ and $\{x^\mu\}$ be local coordinate systems of $\hat{M}$ and Σ, respectively [2]. The embedding map in terms of these coordinates is described by the functions $y^A = \phi^A(x)$. Of course, the image of any vector $\mathbf{v} \in T_p\Sigma$ belongs also to the tangent space $T_p\hat{M}$ of the ambient space $\hat{M}$ and, then, can be written in terms of the coordinate basis $\{\partial_A\}$[3]. In particular, we can write $\partial_\alpha = e^A_\alpha \partial_A$, where

$$e^A_\alpha = \frac{\partial \phi^A}{\partial x^\alpha} \tag{2}$$

are the elements of the differential map $d\phi$ written with respect to the given coordinates. The isometric condition (1) expressed relatively to the coordinate bases takes the form

$$g_{\alpha\beta} = e^A_\alpha e^B_\beta \hat{g}_{AB} \tag{3}$$

Let us now consider the normal vector of the hypersurface Σ with respect to $\hat{M}$.It can be obtained, up to orietation, by solving the following equations:

$$\hat{g}(\partial_\alpha, \mathbf{N}) = 0 \Rightarrow e^A_\alpha N_A = 0 \tag{4}$$

$$\hat{g}(\mathbf{N},\mathbf{N}) = \varepsilon = +1 \tag{5}$$

where we are admitting that the normal vector is spacelike ($\varepsilon = +1$, according to our convention). It follows from the injective condition of the differential map that the rank of e^A_α is equal to the dimension of Σ, and as a consequence there is only one (the number of the codimension) linearly independent normal vector satisfying the above equations. Let us denote it by $\mathbf{N}$.

[1] Here we are adopting the following notation: a small bold letter, as for example $\mathbf{v}$, represents a vector of the tangent space of M while a capital bold letter ($\mathbf{V}$) corresponds to a vector of the tangent space of $\hat{M}$.

[2] Throughout capital Latin indices take value in the range (0,1,...,n) while Greek indices run from (0,1,...,n-1).

[3] Here we are using the common notation $\partial_A = \frac{\partial}{\partial y^A}$ and $\partial_\alpha = \frac{\partial}{\partial x^\alpha}$. We shall represent the components of vectors with respect to these bases as v^α and V^A, respectively.

In the present formalism an important concept is that of a projection tensor Π, which maps vectors $\mathbf{V} \in T_p(\hat{M})$ onto the tangent space $T_p(\Sigma)$ of the hypersurface at $p \in \Sigma$. We define Π by

$$\Pi\left(\mathbf{V}\right) = \mathbf{V} - V^{\perp}\mathbf{N} \tag{6}$$

where here we are using the following notation $V^{\perp} = \hat{g}\left(\mathbf{V}, \mathbf{N}\right)$.

It is clear that $\Pi\left(\mathbf{V}\right)$ can be considered as a vector of $T_p\Sigma$. Therefore it can be written in the basis $\{\partial_\alpha\}$. In particular, the projection of $\partial_A \in T_p\hat{M}$ can be written as $\Pi\left(\partial_A\right) = e^{\alpha}_{A}\partial_\alpha$, for some 'vielbein' e^{α}_{A}. As we shall see later, e^{α}_{A} and e^{A}_{α} are related. Using (6) we can write vector ∂_A as

$$\partial_A = e^{\alpha}_{A}\partial_\alpha + N_A\mathbf{N} \tag{7}$$

Note that this relation is valid only at points of Σ, where $\{\partial_\alpha, \mathbf{N}\}$ constitute a local basis of $T_p\left(\hat{M}\right)$.

To determine the relation between the vielbein functions e^{α}_{A} and e^{A}_{α} let us consider the vectors ∂_β and ∂_A, and calculate their inner product $\hat{g}\left(\partial_\beta, \partial_A\right)$. From (7) it is easy to see that $\hat{g}\left(e^{B}_{\beta}\partial_B, \partial_A\right) = \hat{g}\left(\partial_\beta, e^{\alpha}_{A}\partial_\alpha\right)$, which then shows that

$$e^{\alpha}_{A} = \hat{g}_{AB}g^{\alpha\beta}e^{B}_{\beta} \tag{8}$$

Now let us turn our attention to the decomposition of an arbitrary vector $\mathbf{V} \in T_p\hat{M}$. We have $\mathbf{V} = V^A\ \partial_A$ and from (7) it may be written as a sum of parallel and orthogonal components with respect to the hypersurface Σ:

$$\mathbf{V} = V^{\alpha}\partial_\alpha + V^{\perp}\mathbf{N} \tag{9}$$

where $V^{\alpha} = e^{\alpha}_{A}V^{A}$ corresponds to the coordinates of the projected vector, $\Pi\left(\mathbf{V}\right)$, in the basis $\{\partial_\alpha\}$. It is also useful to express explicitly the relation between the components of the vector in the bases $\{\partial_A\}$ and $\{\partial_\alpha, \mathbf{N}\}$:

$$V^{A} = e^{A}_{\alpha}V^{\alpha} + V^{\perp}N^{A} \tag{10}$$

Now, with the help of the projection operator Π, tensors fields of any rank can be decomposed with respect to the basis $\{\partial_\alpha, \mathbf{N}\}$ at points of Σ. In particular, using (7), the second rank tensor $\mathbf{T} = T^{AB}\partial_A \otimes \partial_B$ can be written as

$$\mathbf{T} = T^{AB}\partial_A \otimes \partial_B = T^{\alpha\beta}\partial_\alpha \otimes \partial_\beta + \psi^{\alpha}\partial_\alpha \otimes \mathbf{N} + \chi^{\beta}\mathbf{N} \otimes \partial_\beta + \phi\mathbf{N} \otimes \mathbf{N} \tag{11}$$

where

$$T^{\alpha\beta} = e^{\alpha}_{A} e^{\beta}_{B} T^{AB} \tag{12}$$

$$\psi^{\alpha} = e^{\alpha}_{A} N_B T^{AB} \tag{13}$$

$$\chi^{\beta} = e^{\beta}_{B} N_A T^{AB} \tag{14}$$

$$\phi = N_A N_B T^{AB} \tag{15}$$

Notice that the components $T^{\alpha\beta}$ are just the components of the projection of $\mathbf{T}$ with respect to the intrinsic coordinates basis $\{\partial_\alpha\}$ of the tangent space of the hypersurface Σ. Clearly, the components of $\mathbf{T}$ in both coordinate bases are related by the equation

$$T^{AB} = e^{A}_{\alpha} e^{B}_{\beta} T^{\alpha\beta} + e^{A}_{\alpha} N^{B} \psi^{\alpha} + N^{A} e^{B}_{\beta} \chi^{\beta} + N^{A} N^{B} \phi \tag{16}$$

We now turn to the question of how differential operators acting on Σ and $\hat{M}$ are related. Let D and ∇ denote the covariant derivative relative to the metrics $\hat{g}$ and g respectively. In the case of an isometric embedding ∇ corresponds to the induced covariant derivative [39]:

$$\nabla_{\mathbf{w}} \mathbf{v} = \Pi \left(D_{\mathbf{w}} \mathbf{v} \right) \tag{17}$$

where now $\mathbf{v}$ and $\mathbf{w}$ are two vectors fields of the tangent space of Σ. (It is important to note that in the expression $D_{\mathbf{w}}\mathbf{v}$, instead of $\mathbf{v}$ and $\mathbf{w}$, we should consider any extensions of these vector fields [39]).

Now take $\mathbf{w} = \partial_\beta$ and write $\mathbf{v}$ in the basis $\{\partial_A\}$. From (17) we obtain

$$\nabla_\beta v^{\alpha} = e^{\alpha}_{A} e^{B}_{\beta} D_B v^{A} \tag{18}$$

Another important concept in the context of the embedding formalism is that of extrinsic curvature K, which can be defined from the orthogonal components of the vector $D_{\mathbf{w}}\mathbf{v}$ as

$$K \left(\mathbf{v}, \mathbf{w} \right) \equiv \hat{g} \left(D_{\mathbf{w}} \mathbf{v}, \mathbf{N} \right) = -\hat{g} \left(\mathbf{v}, D_{\mathbf{w}} \mathbf{N} \right) \tag{19}$$

(The latter equality comes from the condition of compatibility between $\hat{g}$ and the covariant derivative D). It is easy to see that the extrinsic curvature is a symmetric tensor whose components in the coordinates $\{\partial_\alpha\}$ can be written as

$$K_{\alpha\beta} = -e^{A}_{\alpha} e^{B}_{\beta} D_B N_A \tag{20}$$

Now let us consider the covariant derivative of the normal vectors $\mathbf{N}$ along tangent directions of Σ. Relatively to the basis $\{\partial_\alpha, \mathbf{N}\}$, we find the following decomposition:

$$D_\beta \mathbf{N} = -K^\alpha_\beta \partial_\alpha \tag{21}$$

Now we have all the necessary ingredients to analyze the decomposition of the covariant derivative $D_B\mathbf{V}$ of an arbitrary vector field $\mathbf{V}$ of the tangent space $\hat{M}$. Using (7), we can split the covariant derivative in two terms: one related to derivative along the tangent directions of Σ and the other one, related to the normal direction:

$$D_B\mathbf{V} = e^\beta_B D_\beta \mathbf{V} + N_B D_\perp \mathbf{V} \tag{22}$$

where we have introduced the following notation: $D_\perp \mathbf{V} = D_\mathbf{N}\mathbf{V}$. As the vector $\mathbf{V}$ itself can be also decomposed according to (9), then, the first term on the right hand side of (22) gives

$$D_\beta \mathbf{V} = D_\beta \left(V^\alpha \partial_\alpha\right) + D_\beta \left(V^\perp \mathbf{N}\right) \tag{23}$$

Using (17), (19) and (21), it follows that

$$D_\beta \left(V^\alpha \partial_\alpha\right) = \nabla_\beta \left(V^\alpha \partial_\alpha\right) + K_{\alpha\beta} V^\alpha \mathbf{N} \tag{24}$$

$$D_\beta \left(V^\perp \mathbf{N}\right) = \mathbf{N} \nabla_\beta V^\perp - V^\perp K^\alpha_\beta \partial_\alpha \tag{25}$$

Therefore, the derivative of the vector $\mathbf{V}$ along the tangent direction $\Pi\left(\partial_B\right)$ has the following components in the coordinates $\{\partial_A\}$:

$$\begin{aligned} e^\beta_B D_\beta V^A = e^\beta_B \big\{ & e^A_\alpha \left[\nabla_\beta V^\alpha - K^\alpha_\beta V^\perp\right] \\ & + N^A \left[\nabla_\beta V^\perp + K_{\alpha\beta} V^\alpha\right] \big\} \end{aligned} \tag{26}$$

As we have already mentioned, the second term in (22) corresponds to the covariant derivative relative to the normal direction. Thus, collecting all terms, we find

$$\begin{aligned} D_B V^A = {} & e^\beta_B e^A_\alpha \left[\nabla_\beta V^\alpha - V^\perp K^\alpha_\beta\right] \\ & + e^\beta_B N^A \left[\nabla_\beta V^\perp + K_{\alpha\beta} V^\alpha\right] + N_B D_\perp V^A \end{aligned} \tag{27}$$

In similar manner we can obtain the decomposition of the covariant derivative of the tensor $\mathbf{T} = T^{AB} \partial_A \otimes \partial_B$. Thus we have

$$D_C \left(T^{AB} \partial_A \otimes \partial_B\right) = e^\gamma_C D_\gamma \left(T^{AB} \partial_A \otimes \partial_B\right) + N_C D_\perp \left(T^{AB} \partial_A \otimes \partial_B\right) \tag{28}$$

Initially let us we consider the first term on the right-hand side of the above equation:

$$D_\gamma\left(T^{AB}\partial_A\otimes\partial_B\right)=D_\gamma\left(T^{\alpha\beta}\partial_\alpha\otimes\partial_\beta+\psi^\alpha\partial_\alpha\otimes\mathbf{N}+\chi^\beta\mathbf{N}\otimes\partial_\beta+\phi\mathbf{N}\otimes\mathbf{N}\right) \tag{29}$$

Each term on the right-hand side of the latter equation can be analyzed separately:

$$D_\gamma\left(T^{\alpha\beta}\partial_\alpha\otimes\partial_\beta\right)=\nabla_\gamma\left(T^{\alpha\beta}\partial_\alpha\otimes\partial_\beta\right)+T^{\alpha\beta}K_{\alpha\gamma}\mathbf{N}\otimes\partial_\beta+T^{\alpha\beta}K_{\gamma\beta}\partial_\alpha\otimes\mathbf{N} \tag{30}$$

$$\begin{aligned} D_\gamma\left(\psi^\alpha\partial_\alpha\otimes\mathbf{N}\right) &= \nabla_\gamma(\psi^\alpha\partial_\alpha)\otimes\mathbf{N}+\psi^\alpha K_{\alpha\gamma}\mathbf{N}\otimes\mathbf{N} \\ &\quad -\psi^\alpha K^\beta_\gamma\partial_\alpha\otimes\partial_\beta \end{aligned} \tag{31}$$

$$\begin{aligned} D_\gamma\left(\chi^\beta\mathbf{N}\otimes\partial_\beta\right) &= \mathbf{N}\otimes\nabla_\gamma(\chi^\beta\partial_\beta)+\chi^\beta K_{\beta\gamma}\mathbf{N}\otimes\mathbf{N} \\ &\quad -\chi^\beta K^\alpha_\gamma\partial_\alpha\otimes\partial_\beta \end{aligned} \tag{32}$$

$$\begin{aligned} D_\gamma\left(\phi\mathbf{N}\otimes\mathbf{N}\right) &= \left(\nabla_\gamma\phi\right)\mathbf{N}\otimes\mathbf{N}-\phi K^\alpha_\gamma\partial_\alpha\otimes\mathbf{N} \\ &\quad -\phi K^\beta_\gamma\mathbf{N}\otimes\partial_\beta \end{aligned} \tag{33}$$

Thus we have the following:

$$\begin{aligned} e^\gamma_C D_\gamma T^{AB} &= e^\gamma_C\left\{e^A_\alpha e^B_\beta\left[\nabla_\gamma T^{\alpha\beta}-\psi^\alpha K^\beta_\gamma-\chi^\beta K^\alpha_\gamma\right]\right. \\ &\quad + N^A e^B_\beta\left[\varepsilon T^{\alpha\beta}K_{\alpha\gamma}+\nabla_\gamma\chi^\beta-\phi K^\beta_\gamma\right] \\ &\quad + e^A_\alpha N^B\left[T^{\alpha\beta}K_{\gamma\beta}+\nabla_\gamma\psi^\alpha-\phi K^\alpha_\gamma+\psi^\alpha\theta^{cb}_\gamma\right] \\ &\quad \left. + N^A N^B\left[\psi^\alpha K_{\alpha\gamma}+\chi^\beta K_{\beta\gamma}+(\nabla_\gamma\phi)\right]\right\} \end{aligned} \tag{34}$$

With the help of relation (34) and writing the components in second term of (28) as $D_\perp T^{AB}$ we can establish a relation between the components of the covariant derivative of the tensor $\mathbf{T}$ in the bases $\{\partial_A\}$ and $\{\partial_\alpha,\mathbf{N}\}$: $D_C T^{AB}=e^\gamma_C D_\gamma T^{AB}+N^A D_\perp T^{AB}$, where $e^\gamma_C D_\gamma T^{AB}$ is given by (34).

In the next section we are going to discuss the induction of the higher-dimensional electromagnetic equations into the hypersurface Σ. The higher-dimensional equations shall be prescribed as a higher-dimensional generalization of the usual Maxwell equations. In this case, the field equations will involve the divergence of an anti-symmetric second order tensor which will play the role of the electromagnetic field. Then, it is important to know how the divergence of a tensor is treated in the present embedding formalism. So let us take the contraction between the indices A and C in the covariant

derivative of the tensor $\mathbf{T}$. In the local coordinates, we have:

$$D_A T^{AB} = e^{\gamma}_{A} D_{\gamma} T^{AB} + N_A D_{\perp} T^{AB} \tag{35}$$

Using the relation (34) and remembering that $e^A_\alpha e^\gamma_A = \delta^\gamma_\alpha$ and $e^\gamma_A N^A = 0$, we find:

$$\begin{aligned} D_A T^{AB} = & \, e^B_\beta \left[\nabla_\alpha T^{\alpha\beta} - \psi^\alpha K^\beta_\alpha - \chi^\beta K \right] \\ & + N^B \left[T^{\alpha\beta} K_{\gamma\beta} + \nabla_\alpha \psi^\alpha - \phi K \right] + N_A D_\perp T^{AB} \end{aligned} \tag{36}$$

where $K = K^\alpha_\alpha$ is the trace of the extrinsic curvature. Considering the parallel and orthogonal projection of the vector $D_A T^{AB}$ with respect to Σ, we obtain, respectively:

$$\left(D_A T^{AB} \right) e^\beta_B = \nabla_\alpha T^{\alpha\beta} - \psi^\alpha K^\beta_\alpha - \chi^\beta K + W^\beta \tag{37}$$

$$\left(D_A T^{AB} \right) N_B = T^{\alpha\beta} K_{\alpha\beta} + \nabla_\alpha \psi^\alpha - \phi K + Y \tag{38}$$

where $W^\beta = \left(D_\perp T^{AB} \right) N_A e^\beta_B$ and $Y = \left(D_\perp T^{AB} \right) N_A N_B$. It is interesting to notice at this point that χ^α, ψ^α and W^α behave as vectors with respect to the coordinates transformation of Σ, while ϕ and Y are scalar quantities.

3 The induced electromagnetic equations on the hypersurface

In this section we shall investigate how a higher-dimensional electrodynamics defined on the manifold $\hat{M}$ might induces four-dimensional electromagnetism on the manifold Σ, (which we take to be the ordinary space-time). So, let $\mathcal{A}^A$ be a higher-dimensional vector field, which we assume to play the role of a higher-dimensional vector potential defined on $\hat{M}$ and let us consider its projection onto the tangent space $T(\Sigma)$ of Σ:

$$\mathcal{A}^\alpha = e^\alpha_A \, \mathcal{A}^A \tag{39}$$

With the help of (3) and (39) it is not difficult to show that a gauge transformation of the potential, $\mathcal{A}^A \rightarrow \mathcal{A}^A + \hat{g}^{AB} \partial_B f$, induces the gauge transformation of the projected potential $\mathcal{A}^\alpha \rightarrow \mathcal{A}^\alpha + g^{\alpha\beta} \partial_\beta f$.

In order to make contact with four-dimensional electrodynamics let us suppose the existence, in addition to the projected potential $\mathcal{A}^\alpha$, of an "intrinsic" electromagnetic potential A^α confined to the spacetime and that

could be measured by four-dimensional observers living on Σ. It is then reasonable to assume a kind of "isometric" condition for the potential holds, i.e., we assume that the intrinsic potential A^{α} should be identified to $\mathcal{A}^{\alpha}$. That is

$$A^{\alpha} = \mathcal{A}^{\alpha} = e^{\alpha}_{A}\, \mathcal{A}^{A} \tag{40}$$

In analogy with four-dimensional electrodynamics the higher-dimensional electromagnetic tensor $\mathcal{F}_{AB}$ is defined as $\mathcal{F}_{AB} = D_A\mathcal{A}_B - D_B\mathcal{A}_A$, while the intrinsic four-dimensional electromagnetic tensor keeps its usual form $F_{\alpha\beta} = \nabla_{\alpha}A_{\beta} - \nabla_{\beta}A_{\alpha}$. Let us now compare the projection of $\mathcal{F}_{AB}$ relative to the submanifold Σ with the "intrinsic" electromagnetic tensor $F_{\alpha\beta}$. From (27) we can calculate the covariant derivative of the higher-dimensional potential, which, projected into the hypersurface, gives:

$$e^{B}_{\beta}e^{C}_{\gamma}D_B\mathcal{A}_C = \nabla_{\beta}\mathcal{A}_{\gamma} - \mathcal{A}^{\perp}K_{\gamma\beta} = \nabla_{\beta}A_{\gamma} - \mathcal{A}^{\perp}K_{\gamma\beta} \tag{41}$$

Notice that the second equality is obtained from the 'isometric' condition (40), which allows us to substitute $\mathcal{A}_{\gamma}$ by the intrinsic potential A_{γ}. Using the above equation and taking into account that the extrinsic curvature tensor of Σ is symmetric it follows that

$$\mathcal{F}_{\alpha\beta} = e^{A}_{\alpha}e^{B}_{\beta}\mathcal{F}_{AB} = F_{\alpha\beta} \tag{42}$$

In a geometric view of gauge theories we may regard the potential as playing the role of a connection, while the field tensor is the associated curvature tensor. Thus, the above condition relating the projected curvature and the intrinsic curvature may be viewed as the analogue of the Gauss equation which gives a relation of the Riemannian curvature tensors of the embedded manifold and the ambient space.

At this point, let us admit that the electromagnetic field equations in $\hat{M}$ are given by a higher-dimensional version of the Maxwell equations, that is, let us assume that the dynamics of $\mathcal{A}^{A}$ is given by

$$D_A\mathcal{F}^{AB} = g^2J^B \tag{43}$$

where g^2 is the higher-dimensional coupling constant and J^B is a higher-dimensional electric density current that is supposed to satisfy the continuity equation $D_BJ^B = 0$. The higher-dimensional field equation can be decomposed in parallel and orthogonal parts relatively to Σ in much the same way

as was done in (36). Thus, using (36) and (42), the parallel projection yields the induced field equations in terms of the intrinsic electromagnetic field $F^{\alpha\beta}$

$$\nabla_\alpha F^{\alpha\beta} = g^2 J^B e_B^\beta + \psi^\alpha \left(K_\alpha^\beta - \delta_\alpha^\beta K \right) - W^\beta \tag{44}$$

where $W^\beta = \left(D_\perp T^{AB} \right) N_A e_B^\beta$. Note that here we have used the anti-symmetry of the tensor $\mathcal{F}^{AB}$, which allows us to write $\chi^\alpha = -\psi^\alpha$ (see (13) and (14))

The sum of the terms on the right-hand side of the above equation, which behaves as a vector field of Σ with respect to the coordinates transformations of the hypersurface, appears in (44) as an effective density of current. It can be checked that this induced current has the important property of being divergenceless. It is also worthy of mention that the extrinsic curvature of the hypersurface contributes as a source of the four-dimensional electromagnetic field.

Finally, the orthogonal projection of the field equation (43) gives the following constraint for the vector field ψ^α :

$$\nabla_\alpha \psi^\alpha = g^2 J^A N_A \tag{45}$$

4 Existence of embedded solutions of the induced equation

As we have already pointed out, this new formulation of the electromagnetism raises the fundamental question of whether all known solutions of the usual four-dimensional Maxwell equations are contained in the induced equations. We are going to consider this question here following an approach which bears a close analogy with the formulation of the Campbell-Magaard embedding theorem [14].

Let $A^\alpha(x)$ be a given four-dimensional potential and $p \in \Sigma$. We want to show that there exists a solution $\mathcal{A}^A(y)$ of the higher-dimensional Maxwell equations (43) and an open set $U \subset \Sigma$ containing the point p, such that

$$e_A^\alpha \mathcal{A}^A \Big|_U = A^\alpha \tag{46}$$

i.e. the projection of the 5D-solution onto the tangent space $T(\Sigma)$ of the hypersurface is equal to intrinsic potential $A^\alpha(x)$ given.

As far as the existence of solutions is concerned there is no loss of generality if we admit that the field equations in the higher-dimensional space are the sourceless Maxwell equations

$$D_A \mathcal{F}^{AB} = 0 \tag{47}$$

Now, if we prove that it is possible to carry out an "isometric embedding" of any four-dimensional potential A^α into $\hat{M}$, then clearly we have demonstrated that any arbitrary of four-dimensional (physical) current can be simulated, or rather generated, by the embedding mechanism.

Let us start the proof. Since $\mathcal{F}^{AB}$ is an anti-symmetric tensor, it satisfies identically, i.e. independent of any field equation, the condition:

$$D_B D_A \mathcal{F}^{AB} = 0 \tag{48}$$

Using the expression (36) for the anti-symmetric tensor $\mathcal{F}^{AB}$ we get the following identity:

$$D_B \left[e^B_\beta \left(\nabla_\alpha F^{\alpha\beta} - j^\beta \right) + n^B \left(\nabla_\alpha \psi^\alpha \right) \right] = 0 \tag{49}$$

Since the equations (47) and (48) are prescribed in an open set of $\hat{M}$, then equation (49) must be also valid in $\hat{M}$. Therefore, in (49) we must interpret $F^{\alpha\beta}, j^\beta, n^B, \nabla_\alpha \psi^\alpha$ as extensions of these quantities which were originally defined in the hypersurface Σ. In the sequel we shall denote these extensions with a bar. Now, an extension can be obtained by defining a foliation of $\hat{M}$. A foliation adapted to the embedding may be given by considering a family of functions $f^A(x, \ell)$ which depend on the coordinates of Σ and on the extra coordinate ℓ, with the condition

$$f^A(x, 0) = \phi^A(x) \tag{50}$$

where $\phi^A(x)$ is the original embedding map. If additionally we take $f^A(x, \ell)$ to be differentiable at p (at least with the same class of differentiability of $\hat{M}$), then the set of equations

$$y^A = f^A(x, \ell) \tag{51}$$

may be regarded as a legitimate transformation of coordinates in a neighborhood of p; hence the set of variables $\{x, \ell\}$ will constitute a coordinate

system in a neighborhood of $p \in \hat{M}$. Clearly, each equation $\ell = const$ defines a hypersurface of $\hat{M}$. In particular, $\ell = 0$ is the equation that defines the hypersurface Σ by virtue of (50). Naturally the mathematical formalism developed previously is also applicable to any leaf ($\ell = const$) of the foliation, with the simple change

$$e_{\alpha}^{A}(x) \rightarrow \bar{e}_{\alpha}^{A}(x,\ell) = \frac{\partial f^{A}}{\partial x^{\alpha}} \tag{52}$$

Thus, in equation (49) the projected quantities will also depend on the extra coordinate ℓ. When $\ell = 0$ they reduce to the original ones defined on the hypersurface Σ.

Now if we admit that equation (44) holds in a neighborhood of $p \in \hat{M}$, then equation (49) reduces to

$$\bar{n}^{B} D_{B}\left(\bar{\nabla}_{\alpha}\bar{\psi}^{\alpha}\right) - \left(\bar{\nabla}_{\alpha}\bar{\psi}^{\alpha}\right) D_{B}\bar{n}^{B} = 0 \tag{53}$$

The above equation gives the propagation of $\nabla_{\alpha}\psi^{\alpha}$ along the normal direction. If $\nabla_{\alpha}\psi^{\alpha} = 0$ in Σ, then clearly the unique solution of (53) is $\bar{\nabla}_{\alpha}\bar{\psi}^{\alpha} = 0$ in a open set of $\hat{M}$. Therefore, as usual in gauge theories, there is a kind of constraint equation, $\nabla_{\alpha}\psi^{\alpha} = 0$, that is automatically propagated by the real dynamical field equations (44). We might say then that the field equations $D_{A}\mathcal{F}^{AB} = 0$, defined in $\hat{M}$, is equivalent to

$$\bar{\nabla}_{\alpha}\bar{F}^{\alpha\beta} = \bar{j}^{\beta} \qquad \text{in } \hat{M} \tag{54}$$

$$\nabla_{\alpha}\psi^{\alpha} = 0 \qquad \text{in } \Sigma \tag{55}$$

Now let us investigate the existence of solutions of (54). With this purpose let us rewrite the above equation in a more convenient form. We begin by showing that $\bar{W}^{\beta}$ is the only term in equation (54) that involves the second derivative of the potential along the normal direction. In order to verify this we rewrite $\bar{W}^{\beta}$ as the following decomposition:

$$\bar{W}^{\beta} = -D_{\perp}\bar{\psi}^{\beta} - \bar{F}^{\alpha\beta}a_{\alpha} - \bar{\psi}^{\alpha}\bar{e}_{A}^{\beta}\theta_{\alpha}^{A} + \bar{\psi}^{\alpha}\bar{K}_{\alpha}^{\beta} \tag{56}$$

where we define $a^{\alpha} = e_{A}^{\alpha} D_{\perp} N^{A}$ (acceleration of the normal vector $\mathbf{N}$) and $\theta_{\alpha}^{A} = [\mathbf{N}, \partial_{\alpha}]^{A}$ (the Lie derivative of $\mathbf{N}$ and the extended vector fields ∂_{α}). It is clear that a^{α} and θ_{α}^{A} depend on the foliation, which is assumed to be sufficiently smooth. Consider now the term $D_{\perp}\bar{\psi}^{\beta}$. Writing the normal vector in the coordinate basis $\{\partial_{\alpha}, \partial_{\ell}\}$ and using (7), we can show that

$$D_\perp \bar{\psi}^\beta = \mathbf{N}\left(\bar{\psi}^\beta\right) = \frac{1}{N_A \ell^A}\left(\frac{\partial \bar{\psi}^\beta}{\partial \ell} - \ell^A \bar{e}^\alpha_A \partial_\alpha \bar{\psi}^\beta\right) \tag{57}$$

where $\ell^A = \frac{\partial f^A}{\partial \ell}$. It turns out that $\bar{\psi}^\beta$ already involves a normal derivative of the potential, as we can explicitly see by rewriting it as

$$\bar{\psi}^\alpha = g^{\alpha\gamma}\left(\nabla_\gamma \mathcal{A}^\perp - D_\perp \mathcal{A}_\gamma + \mathcal{A}_C \theta^C_\gamma\right) \tag{58}$$

Therefore, as we have mentioned before, $\bar{W}^\beta$ depends on the second derivative of $\mathcal{A}_\gamma$ with respect to the coordinate ℓ. If we isolate this term at the left-hand side, we can put the equation (54) in the following form:

$$\frac{\partial^2 \mathcal{A}_\beta}{\partial \ell^2} = Q_\beta\left(\mathcal{A}_\alpha, \frac{\partial \mathcal{A}_\alpha}{\partial \ell}, x, \ell\right) \tag{59}$$

where Q_β represent functions that are analytic with respect to all the arguments. It is important to bear in mind that the normal component of the potential $\bar{N}^A\ \mathcal{A}_A = \mathcal{A}^\perp$ may be arbitrarily chosen and should be regarded as a known analytical function.

It turns out that according to the Cauchy-Kowalewskaya theorem equation (59) admits analytic solutions and that these solutions are unique as long as the initial conditions $\partial_\ell \mathcal{A}_\beta|_\Sigma$ and $\mathcal{A}_\alpha|_\Sigma$ are specified [41]. Moreover, the initial functions can be any arbitrary analytic functions. Thus, we might choose $\mathcal{A}_\alpha|_\Sigma = A_\alpha$, in order to satisfy the isometric embedding condition (40). In addition, we might select functions $\partial_\ell \mathcal{A}_\beta|_\Sigma$ that satisfy the constraint equation, whose solution always exists. Finally, with the solution of (59), we are able to construct a 5D-potential $\mathcal{A}^A = \bar{e}^A_\alpha\ \mathcal{A}^\alpha +\ \mathcal{A}^\perp \bar{N}^A$ which clearly satisfies the vacuum 5D-Maxwell equations (47). This completes the proof that the set of all analytic solutions of the four-dimensional Maxwell equations is contained in the five-dimensional electromagnetism. This generalizes our previous study in this subject in which we have obtained the same result investigating the solutions of the field equations in a flat ambient space [42].

5 The effective field equations in a brane

In this section we are going to start by admitting that matter is localized in a brane Σ by some confinement mechanism [24]. Our purpose here is to

investigate the effects of the concentration of charges and currents on the field equations for the electromagnetic field in the brane.

The confined 4D-current can be described by means of a Dirac delta function with support on the hypersurface. Using a foliation adapted to the embedding, like the foliation given in (51), the bulk current density J^B can be written as:

$$J^B = e^B_\beta j^\beta \frac{\delta(\ell)}{N_A \ell^A} \tag{60}$$

where j^β describes the current density of the localized charges in Σ.

It is reasonable to expect that, by virtue of the concentration of the electric charge in Σ, the first normal derivative of the bulk potential is discontinuous in Σ and that the discontinuity through Σ should depend on j^β. To determine the amount of this jump, let us integrate the equation (54) in some finite region of the bulk contained in the range $-\varepsilon < \ell < \varepsilon$. Taking into account that the 'volume' element of the bulk is $\sqrt{|\bar{g}|} N_A \ell^A d\ell dx^1 ... dx^n$ (in the coordinates adapted to the foliation) and isolating the source term on the right-hand side, we obtain from (44):

$$\underset{-\varepsilon<\ell<\varepsilon}{\int ... \int} \left\{ \left[\bar{W}^\beta - \bar{\psi}^\alpha \left(\bar{K}^\beta_\alpha - \delta^\beta_\alpha \bar{K} \right) \right] + \bar{\nabla}_\alpha \bar{F}^{\alpha\beta} \right\} \sqrt{|\bar{g}|} N_A \ell^A d\ell dx^1 ... dx^n =$$

$$= g^2 \underset{\ell=0}{\int ... \int} j^\beta \sqrt{|g|} dx^1 ... dx^n \tag{61}$$

All the terms inside the integral on the left hand side of equation (61) are bounded functions, except $\bar{W}^\beta$ which contains a second derivative of the bulk potential with respect to the normal direction, as we have seen in the previous section. Therefore $\bar{W}^\beta$ might contain a singular distribution. Now taking the limit $\varepsilon \to 0$, we find, using (56) and (57), that the equation (61) yields

$$\left[\psi^\beta \right] = -g^2 j^\beta \tag{62}$$

where $\left[\psi^\beta \right] = \psi^\beta (\ell \to 0^+) - \psi^\beta (\ell \to 0^-)$. If we admit, as usual, that the brane has Z_2 reflection symmetry then it is reasonable to assume that ψ^α is anti-symmetric with respect to Σ. Thus it follows that

$$\psi^\beta_{(0^+)} = -\psi^\beta_{(0^-)} = -\frac{g^2}{2} j^\beta \tag{63}$$

Now remember that the extrinsic curvature is related to the matter content in the brane according to the equation[4]:

$$K^{+}_{\mu\nu} = -K^{-}_{\mu\nu} = \frac{1}{2}G_5\left(S_{\mu\nu} - \frac{1}{3}g_{\mu\nu}S\right) \tag{64}$$

where the 4D-energy-momentum tensor $S_{\mu\nu}$ can be written as a sum of two terms: one related to the tension of the brane and the other corresponding to the ordinary energy distribution in the brane

$$S_{\mu\nu} = -\lambda g_{\mu\nu} + \tau_{\mu\nu} \tag{65}$$

It is interesting to notice that ψ^{β} and $K_{\mu\nu}$ are not well defined in the brane, since they assume different values in Σ as we approach the hypersurface from above ($\ell > 0$) or from bellow ($\ell < 0$). However, the product $\psi^{\beta}K_{\mu\nu}$ is well defined in the brane. Hence the equation (44) gives a well defined equation for the electromagnetic tensor $F^{\alpha\beta}$ in Σ when we take the limit $\ell \to 0$:

$$\nabla_{\alpha}F^{\alpha\beta} = \frac{1}{4}g^2G_5\lambda j^{\beta} - \frac{1}{4}g^2G_5 j^{\alpha}\tau^{\beta}_{\alpha} - W^{\beta}_{reg} \tag{66}$$

where W^{β}_{reg} is the regular part of $\bar{W}^{\beta}$, which is obtained in the limit. Thus we can consider that equation (66) represents the effective equation which governs the electromagnetic field as seen by 4D-observers. Of course (66) must fulfill an important requirement if it is to be considered a good field equation in the brane. It should be equivalent to the Maxwell equations in the low-energy regime in order to be consistent with the successful description of the electromagnetic phenomena by the 4D-Maxwell equations. As direct consequence of this requirement, it is necessary that the unknown coupling constants must obey the constraint $g^2G_5\lambda = 4e^2$. It follows then that the effective equations assume the form

$$\nabla_{\alpha}F^{\alpha\beta} = e^2 j^{\beta} - \frac{e^2}{\lambda}j^{\alpha}\tau^{\beta}_{\alpha} - W^{\beta}_{reg} \tag{67}$$

It is obvious that, compared to the Maxwell equations, the effective equations have two additional terms which contribute as sources of the electromagnetic field. Let us make some comments on the interpretation of these terms and estimate their order of magnitude.

[4]The positive sign is in accordance with our definition of the extrinsic curvature tensor (19)

It is generally accepted that the the order of the brane tension λ is much greater than the energy scale of the ordinary matter whose distribution is described by $\tau_{\mu\nu}$. So the second term is very small compared to the first one in the low-energy regime. Despite of its tiny effects in the ordinary energy scale, let us try to understand how it works. From the point of view of 4D-observers, the origin of $\frac{e^2}{\lambda} j^\alpha \tau_\alpha^\beta$ might be connected to a unusual coupling between charges and fields (including the electromagnetic field itself since $\tau_{\mu\nu}$ also contains the energy distribution of the electromagnetic field produces by j^β). In a sense, this term behaves as a kind of medium polarization, but with uncommon features. It does not vanishes even when there is no medium at all and the charges are just spread in the vacuum. Another peculiarity is the fact that its effects are restricted to the region where the source is present $\left(j^\beta \neq 0\right)$. To make more precise this interpretation, let us write the density current as $j^\beta = \rho_0 U^\beta$, where ρ_0 is the proper charge density and U^β corresponds to the quadrivelocity of the charges ($U^a U_\alpha = -1$). Taking the inner product with U_β, we can see that the term $\frac{e^2}{\lambda} j^\alpha \tau_\alpha^\beta$ produces a change in the effective value of the charge of the source in a very similar way of a genuine polarization. Indeed, we find

$$\rho_{eff} = -U_\beta \left(j^\beta - \frac{1}{\lambda} j^\alpha \tau_\alpha^\beta \right) = \rho_0 \left(1 + \frac{u}{\lambda} \right) \tag{68}$$

where $u = \tau^{\alpha\beta} U_\alpha U_\beta$ is the energy density as measured by a co-moving inertial frame relative to the charges. If equation (67) does not allow solutions with negative energy distribution, then we can conclude that due to the 'polarization' the effective charge appears to be greater than the real charge. Therefore, this ´polarization' has an opposite sign in comparison with a common polarization of a material medium.

The third term is a vector field W_{reg}^β, which might not be put in correspondence with any known field by the 4D-observers. The reason is that it depends on the behavior of the gauge field in the vicinity of Σ. However, roughly speaking, it could be considered as the effect of a strange vacuum polarization in the brane, although this vacuum polarization would have no connection with the quantum fluctuation of the vacuum. Now let us estimate the magnitude of W_{reg}^β. As we shall see it requires a more careful examination. First, taking the divergence of the effective equation, we find that W^β satisfies the condition

$$\nabla_\beta W_{reg}^\beta = -\frac{e^2}{\lambda} \tau_\alpha^\beta \nabla_\beta j^\alpha \tag{69}$$

It is clear that the divergence of W^{β} has the same order of the second term in equation (67) and, thus can be neglected when compared to $e^2 j^{\alpha}$. However, the divergenceless part of W^{β} cannot be estimated by 4D-observers. This is because the induced equations do not constitute a complete set of equations for the 4D-fields. They can be solved only if they are considered as part of the five-dimensional field equations. However, some analyses regarding specific geometric configurations show that the term cannot be discarded, and, as consequence the 4D-Green function cannot be recovered in the brane at low-energy regime [30, 31].

This negative result gives a clear indication that massless vectors fields cannot be localized in the hypersurface by means of gravity in this scenario. However, as we have already mentioned there are some alternative models which propose other different ways to guarantee the localization of massless gauge field. Here we want to concentrate our discussion on the DGP model.

According to DGP model, the interaction between bulk gauge fields and localized matter in the brane induces corrections in the original Lagrangean due to quantum effects. It is argued that these additional terms represent interactions which are already four-dimensional. For example, in the case of electromagnetism, the total Lagrangean gains an extra term which is exactly $\frac{1}{e^2}F_{\alpha\beta}F^{\alpha\beta}$, the usual 4D-electromagnetic Lagrangean, restricted to the world volume of the brane [38]. Of course, the presence of this extra term will modify the field equations. Let us see that this alteration is easily implemented within our formalism. Indeed, as we have mentioned $F_{\alpha\beta}F^{\alpha\beta}$ is already confined to the brane, as the original source j^{β}. Thus, $F_{\alpha\beta}F^{\alpha\beta}$ can be readily incorporated into the field equations if we take it as part of the bulk current J^B. Thus the new bulk current now becomes:

$$J^B = e^B_{\beta}\left(j^{\beta} - \frac{1}{e^2}\nabla_{\alpha}F^{\alpha\beta}\right)\frac{\delta(\ell)}{N_A \ell^A} \tag{70}$$

Following the previous procedure, it can be readily seen that the discontinuity of the $\bar{\psi}^{\beta}$ through Σ is now given by:

$$\left[\psi^{\beta}\right] = -g^2\left(j^{\beta} - \frac{1}{e^2}\nabla_{\alpha}F^{\alpha\beta}\right) \tag{71}$$

Taking the equation (44) in the limit $\ell \rightarrow 0$ and admitting the reflection symmetry again we obtain the effective field equation in the brane in DGP

scenario:

$$\left\{\left(1+\frac{1}{4}\frac{g^{2}\lambda G_{5}}{e^{2}}\right)\delta_{\gamma}^{\beta}-\frac{1}{4}\frac{g^{2}G_{5}}{e^{2}}\tau_{\gamma}^{\beta}\right\}\nabla_{\alpha}F^{\alpha\gamma}=\frac{1}{4}g^{2}\lambda G_{5}j^{\beta}-\frac{1}{4}g^{2}G_{5}j^{\alpha}\tau_{\alpha}^{\beta}-W_{reg}^{\beta} \tag{72}$$

As in the previous case, in order that equation (72) can give an appropriate low-energy limit, it is necessary that the coupling constant be constraint according to this formula

$$g^{2}\lambda G_{5}=4\sigma e^{2} \tag{73}$$

where σ is some dimensionless constant (in our units). Substituting this constraint into the (72), the effective field equations assume the following form:

$$\left[\delta_{\gamma}^{\beta}-\frac{\sigma}{(1+\sigma)\,\lambda}\tau_{\gamma}^{\beta}\right]\nabla_{\alpha}F^{\alpha\gamma}=\frac{\sigma e^{2}}{(1+\sigma)}j^{\beta}-\frac{\sigma e^{2}}{(1+\sigma)\,\lambda}j^{\alpha}\tau_{\alpha}^{\beta}-\frac{1}{(1+\sigma)}W_{reg}^{\beta} \tag{74}$$

There are two terms that depend on the brane tension λ directly. As we have pointed out before, they are very small compared to first term ($e^{2}j^{\beta}$) at the ordinary scale of energy. So let us focus our analysis on W_{reg}^{β}. Our first remark is that now, in the DGP scenario, the vector field is constrained. Indeed, comparing equation (71) and (74), we find that:

$$W_{reg}^{\beta}=-e^{2}j^{\beta}+(1+\sigma)\left(\delta_{\gamma}^{\beta}-\frac{1}{2\lambda}\tau_{\gamma}^{\beta}\right)\frac{e^{2}}{g^{2}}\left[\psi^{\gamma}\right] \tag{75}$$

This equation shows that there is now two different characteristic parameters: the brane tension λ and the electromagnetic coupling constant g^{2}. We have already investigated the limit of the effective equations in comparison with the energy scale of λ. Now let us consider the other parameter g^{2}. If we admit that the lenght scale $(G_{5}\lambda)^{-1}$ is much smaller than $r_{c}\equiv g^{2}/e^{2}$, which is the most interesting case, then, in the low-energy limit (compared to the brane tension), the above equation reduces to

$$W_{reg}^{\beta}=-e^{2}j^{\beta}+(1+\sigma)\frac{e^{2}}{g^{2}}\left[\psi^{\beta}\right] \tag{76}$$

By the definition of W^{β} and ψ^{β}, we can conclude from (76), that the solution (the bulk potential) will depend on the parameter g^{2}, i.e., $\mathcal{A}^{\beta}\left(x,\ell,g^{2}\right)$. Now if the dependence on g^{2} is such that $\left[\psi^{\beta}\right]\sim\mathcal{O}\left[\left(g^{2}\right)^{-n}\right]$, where $n>-1$,

or even logarithmic, then for large values of g^2, we have approximately $W^{\beta}_{reg} = -e^2 j^{\beta}(x)$, (remember that $j^{\beta}(x)$ does not depend on g^2). It follows then, from (74), that Maxwell equations will be recovered in the brane in the range $(G_5\lambda)^{-1} << r << r_c$.

However, for large distance compared to r_c, it is reasonable to think that the term $\frac{1}{r_c}\left[\psi^{\beta}\right]$ becomes more relevant, and hence the electromagnetic field in the brane in that domain will deviate significantly from its usual behavior in four-dimensional spacetime. In other words, the five-dimensional character of the fundamental electromagnetic field will be appreciable at large distance compared to the scale established by the parameter r_c. It is well known that long range effects are connected to low frequency modes. Therefore, the deviation of the four-dimensional behavior is interpreted as a consequence of the leakage of infrared modes of the field to the extra dimension. This effect is known as infrared transparency and it is a characteristic feature of the DGP model.

6 Final remarks

Originally the DGP model was conceived to deal with the issue of localization of gravitons in brane worlds. According to this model, the Newtonian gravitational potential can be recovered in a brane even if the brane is embedded in a bulk with flat extra dimension. The key element, as the authors realized, is the fact that matter localized in the brane interacting with bulk gauge fields gives rise to a kinetic term for the gauge fields which is constrained to the world volume of the brane. Soon afterwards, this scheme was applied to other gauge interactions as a generic mechanism of localization of massless gauge fields. A curious feature of the DGP model is that deviation of the four-dimensional behavior of the localized field will be manifest at ultra-large distance.

In this paper we have considered an extended version of DGP braneworld scenario and investigated the question of how the electromagnetism is described in the brane. Using a covariant embedding formalism, we have found the effective field equations that dictated the behavior of the electromagnetism field as seen from 4D-observers. The adopted formulation here seems to have some advantages over some other approaches: First, it allows us to treat the issue with great generality since we do not put restrictions on the geometry of neither the bulk nor the brane. Second, based on a covariant

formulation, the interpretation of the terms that appear in the effective field equations are independent of coordinates choices.

In order to keep the maximum generality we have also considered branes whose tension is not necessarily null. Hence, the effective field equations have two parameter which establish two different length scales. Finally, we have determined the condition under which the effective field equations can recover Maxwell equations.

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